Texas
Educator Certification
Examination Program

# Preparation Manual
# Mathematics 7–12 (235)

**Overview and Exam Framework**

**Reference Materials**

**Sample Selected-Response Questions**

**Sample Selected-Response Answers and Rationales**

# Section 3: Overview and Exam Framework Mathematics 7–12 (235)

## Exam Overview

| | |
|---|---|
| **Exam Name** | **Mathematics 7–12** |
| **Exam Code** | 235 |
| **Time** | 5 hours |
| **Number of Questions** | 100 selected-response questions |
| **Format** | Computer-administered test (CAT) |

The TExES Mathematics 7–12 (235) exam is designed to assess whether an examinee has the requisite knowledge and skills that an entry-level educator in this field in Texas public schools must possess. The 100 selected response questions are based on the Mathematics 7–12 exam framework. The exam may contain questions that do not count toward the score. Your final scaled score will be based only on scored questions.

## The Standards

| | |
|---|---|
| **Standard I** | Number Concepts: The mathematics teacher understands and uses numbers, number systems and their structure, operations and algorithms, quantitative reasoning and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) to prepare students to use mathematics. |
| **Standard II** | Patterns and Algebra: The mathematics teacher understands and uses patterns, relations, functions, algebraic reasoning, analysis and technology appropriate to teach the statewide curriculum (TEKS) to prepare students to use mathematics. |
| **Standard III** | Geometry and Measurement: The mathematics teacher understands and uses geometry, spatial reasoning, measurement concepts and principles and technology appropriate to teach the statewide curriculum (TEKS) to prepare students to use mathematics. |
| **Standard IV** | Probability and Statistics: The mathematics teacher understands and uses probability and statistics, their applications and technology appropriate to teach the statewide curriculum (TEKS) to prepare students to use mathematics. |
| **Standard V** | Mathematical Processes: The mathematics teacher understands and uses mathematical processes to reason mathematically, to solve mathematical problems, to make mathematical connections within and outside of mathematics and to communicate mathematically. |

**Standard VI**

Mathematical Perspectives: The mathematics teacher understands the historical development of mathematical ideas, the relationship between society and mathematics, the structure of mathematics and the evolving nature of mathematics and mathematical knowledge.

**Standard VII**

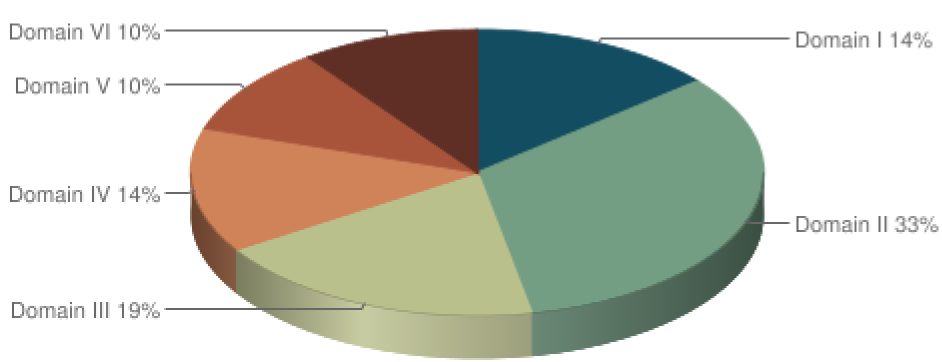
Mathematical Learning and Instruction: The mathematics teacher understands how children learn and develop mathematical skills, procedures and concepts; knows typical errors students make; and uses this knowledge to plan, organize and implement instruction to meet curriculum goals and to teach all students to understand and use mathematics.

**Standard VIII**

Mathematical Assessment: The mathematics teacher understands assessment, and uses a variety of formal and informal assessment techniques appropriate to the learner on an ongoing basis to monitor and guide instruction and to evaluate and report student progress.

## Domains and Competencies

| Domain | Domain Title | Approx. Percentage of Exam | Standards Assessed |
|---|---|---|---|
| I | Number Concepts | 14% | Mathematics 7–12 I |
| II | Patterns and Algebra | 33% | Mathematics 7–12 II |
| III | Geometry and Measurement | 19% | Mathematics 7–12 III |
| IV | Probability and Statistics | 14% | Mathematics 7–12 IV |
| V | Mathematical Processes and Perspectives | 10% | Mathematics 7–12 V–VI |
| VI | Mathematical Learning, Instruction and Assessment | 10% | Mathematics 7–12 VII–VIII |



The content covered by this exam is organized into broad areas of content called **domains**. Each domain covers one or more of the educator standards for this field. Within each domain, the content is further defined by a set of **competencies**. Each competency is composed of two major parts:

- The **competency statement**, which broadly defines what an entry-level educator in this field in Texas public schools should know and be able to do.
- The **descriptive statements**, which describe in greater detail the knowledge and skills eligible for testing.

## Domain I—Number Concepts

**Competency 001—The teacher understands the real number system and its structure, operations, algorithms and representations.**

The beginning teacher:

A. Understands the concepts of place value, number base and decimal representations of real numbers.

B. Understands the algebraic structure and properties of the real number system and its subsets (e.g., real numbers as a field, integers as an additive group).

C. Describes and analyzes properties of subsets of the real numbers (e.g., closure, identities).

D. Selects and uses appropriate representations of real numbers (e.g., fractions, decimals, percents, roots, exponents, scientific notation) for particular situations.

E. Uses a variety of models (e.g., geometric, symbolic) to represent operations, algorithms and real numbers.

F. Uses real numbers to model and solve a variety of problems.

G. Uses deductive reasoning to simplify and justify algebraic processes.

H. Demonstrates how some problems that have no solution in the integer or rational number systems have solutions in the real number system.

**Competency 002—The teacher understands the complex number system and its structure, operations, algorithms and representations.**

The beginning teacher:

A. Demonstrates how some problems that have no solution in the real number system have solutions in the complex number system.

B. Understands the properties of complex numbers (e.g., complex conjugate, magnitude/modulus, multiplicative inverse).

C. Understands the algebraic structure of the complex number system and its subsets (e.g., complex numbers as a field, complex addition as vector addition).

D. Selects and uses appropriate representations of complex numbers (e.g., vector, ordered pair, polar, exponential) for particular situations.

E. Describes complex number operations (e.g., addition, multiplication, roots) using symbolic and geometric representations.

**Competency 003—The teacher understands number theory concepts and principles and uses numbers to model and solve problems in a variety of situations.**

The beginning teacher:

A. Applies ideas from number theory (e.g., prime numbers and factorization, the Euclidean algorithm, divisibility, congruence classes, modular arithmetic, the fundamental theorem of arithmetic) to solve problems.

B. Applies number theory concepts and principles to justify and prove number relationships.

C. Compares and contrasts properties of vectors and matrices with properties of number systems (e.g., existence of inverses, non-commutative operations).

D. Uses properties of numbers (e.g., fractions, decimals, percents, ratios, proportions) to model and solve real-world problems.

E. Applies counting techniques such as permutations and combinations to quantify situations and solve problems.

F. Uses estimation techniques to solve problems and judges the reasonableness of solutions.

## Domain II—Patterns and Algebra

**Competency 004—The teacher uses patterns to model and solve problems and formulate conjectures.**

The beginning teacher:

A. Recognizes and extends patterns and relationships in data presented in tables, sequences or graphs.

B. Uses methods of recursion and iteration to model and solve problems.

C. Uses the principle of mathematical induction.

D. Analyzes the properties of sequences and series (e.g., Fibonacci, arithmetic, geometric) and uses them to solve problems involving finite and infinite processes.

E. Understands how sequences and series are applied to solve problems in the mathematics of finance (e.g., simple, compound and continuous interest rates; annuities).

**Competency 005—The teacher understands attributes of functions, relations and their graphs.**

The beginning teacher:

A. Understands when a relation is a function.

B. Identifies the mathematical domain and range of functions and relations and determines reasonable domains for given situations.

C. Understands that a function represents a dependence of one quantity on another and can be represented in a variety of ways (e.g., concrete models, tables, graphs, diagrams, verbal descriptions, symbols).

D. Identifies and analyzes even and odd functions, one-to-one functions, inverse functions and their graphs.

E. Applies basic transformations [e.g., $k\,f(x)$, $f(x) + k$, $f(x - k)$, $f(kx)$, $|f(x)|$] to a parent function, $f$, and describes the effects on the graph of $y = f(x)$.

F. Performs operations (e.g., sum, difference, composition) on functions, finds inverse relations and describes results symbolically and graphically.

G. Uses graphs of functions to formulate conjectures of identities [e.g., $y = x^2 - 1$ and $y = (x - 1)(x + 1)$, $y = \log x^3$ and $y = 3 \log x$, $y = \sin(x + \frac{\pi}{2})$ and $y = \cos x$].

**Competency 006—The teacher understands linear and quadratic functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.**

The beginning teacher:

A. Understands the concept of slope as a rate of change and interprets the meaning of slope and intercept in a variety of situations.

B. Writes equations of lines given various characteristics (e.g., two points, a point and slope, slope and $y$-intercept).

C. Applies techniques of linear and matrix algebra to represent and solve problems involving linear systems.

D. Analyzes the zeros (real and complex) of quadratic functions.

E. Makes connections between the $y = ax^2 + bx + c$ and the $y = a(x - h)^2 + k$ representations of a quadratic function and its graph.

F. Solves problems involving quadratic functions using a variety of methods (e.g., factoring, completing the square, using the quadratic formula, using a graphing calculator).

G. Models and solves problems involving linear and quadratic equations and inequalities using a variety of methods, including technology.

**Competency 007—The teacher understands polynomial, rational, radical, absolute value and piecewise functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.**

The beginning teacher:

A. Recognizes and translates among various representations (e.g., written, tabular, graphical, algebraic) of polynomial, rational, radical, absolute value and piecewise functions.

B. Describes restrictions on the domains and ranges of polynomial, rational, radical, absolute value and piecewise functions.

C. Makes and uses connections among the significant points (e.g., zeros, local extrema, points where a function is not continuous or not differentiable) of a function, the graph of the function and the function's symbolic representation.

D. Analyzes functions in terms of vertical, horizontal and slant asymptotes.

E. Analyzes and applies the relationship between inverse variation and rational functions.

F. Solves equations and inequalities involving polynomial, rational, radical, absolute value and piecewise functions using a variety of methods (e.g., tables, algebraic methods, graphs, use of a graphing calculator) and evaluates the reasonableness of solutions.

G. Models situations using polynomial, rational, radical, absolute value and piecewise functions and solves problems using a variety of methods, including technology.

**Competency 008—The teacher understands exponential and logarithmic functions, analyses their algebraic and graphical properties and uses them to model and solve problems.**

The beginning teacher:

A. Recognizes and translates among various representations (e.g., written, numerical, tabular, graphical, algebraic) of exponential and logarithmic functions.

B. Recognizes and uses connections among significant characteristics (e.g., intercepts, asymptotes) of a function involving exponential or logarithmic expressions, the graph of the function and the function's symbolic representation.

C. Understands the relationship between exponential and logarithmic functions and uses the laws and properties of exponents and logarithms to simplify expressions and solve problems.

D. Uses a variety of representations and techniques (e.g., numerical methods, tables, graphs, analytic techniques, graphing calculators) to solve equations, inequalities and systems involving exponential and logarithmic functions.

E. Models and solves problems involving exponential growth and decay.

F. Uses logarithmic scales (e.g., Richter, decibel) to describe phenomena and solve problems.

G. Uses exponential and logarithmic functions to model and solve problems involving the mathematics of finance (e.g., compound interest).

H. Uses the exponential function to model situations and solve problems in which the rate of change of a quantity is proportional to the current amount of the quantity [i.e., $f'(x) = kf(x)$].

**Competency 009—The teacher understands trigonometric and circular functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.**

The beginning teacher:

A. Analyzes the relationships among the unit circle in the coordinate plane, circular functions and the trigonometric functions.

B. Recognizes and translates among various representations (e.g., written, numerical, tabular, graphical, algebraic) of trigonometric functions and their inverses.

C. Recognizes and uses connections among significant properties (e.g., zeros, axes of symmetry, local extrema) and characteristics (e.g., amplitude, frequency, phase shift) of a trigonometric function, the graph of the function and the function's symbolic representation.

D. Understands the relationships between trigonometric functions and their inverses and uses these relationships to solve problems.

E. Uses trigonometric identities to simplify expressions and solve equations.

F. Models and solves a variety of problems (e.g., analyzing periodic phenomena) using trigonometric functions.

G. Uses graphing calculators to analyze and solve problems involving trigonometric functions.

**Competency 010—The teacher understands and solves problems using differential and integral calculus.**

The beginning teacher:

A. Understands the concept of limit and the relationship between limits and continuity.

B. Relates the concept of average rate of change to the slope of the secant line and relates the concept of instantaneous rate of change to the slope of the tangent line.

C. Uses the first and second derivatives to analyze the graph of a function (e.g., local extrema, concavity, points of inflection).

D. Understands and applies the fundamental theorem of calculus and the relationship between differentiation and integration.

E. Models and solves a variety of problems (e.g., velocity, acceleration, optimization, related rates, work, center of mass) using differential and integral calculus.

F. Analyzes how technology can be used to solve problems and illustrate concepts involving differential and integral calculus.

## Domain III—Geometry and Measurement

**Competency 011—The teacher understands measurement as a process.**

The beginning teacher:

A. Applies dimensional analysis to derive units and formulas in a variety of situations (e.g., rates of change of one variable with respect to another) and to find and evaluate solutions to problems.

B. Applies formulas for perimeter, area, surface area and volume of geometric figures and shapes (e.g., polygons, pyramids, prisms, cylinders, cones, spheres) to solve problems.

C. Recognizes the effects on length, area or volume when the linear dimensions of plane figures or solids are changed.

D. Applies the Pythagorean theorem, proportional reasoning and right triangle trigonometry to solve measurement problems.

E. Relates the concept of area under a curve to the limit of a Riemann sum.

F. Uses integral calculus to compute various measurements associated with curves and regions (e.g., area, arc length) in the plane, and measurements associated with curves, surfaces and regions in three-space.

**Competency 012—The teacher understands geometries, in particular Euclidian geometry, as axiomatic systems.**

The beginning teacher:

A. Understands axiomatic systems and their components (e.g., undefined terms, defined terms, theorems, examples, counterexamples).

B. Uses properties of points, lines, planes, angles, lengths and distances to solve problems.

C. Applies the properties of parallel and perpendicular lines to solve problems.

D. Uses properties of congruence and similarity to explore geometric relationships, justify conjectures and prove theorems.

E. Describes and justifies geometric constructions made using compass and straightedge, reflection devices and other appropriate technologies.

F. Demonstrates an understanding of the use of appropriate software to explore attributes of geometric figures and to make and evaluate conjectures about geometric relationships.

G. Compares and contrasts the axioms of Euclidean geometry with those of non-Euclidean geometry (i.e., hyperbolic and elliptic geometry).

**Competency 013—The teacher understands the results, uses and applications of Euclidian geometry.**

The beginning teacher:

A. Analyzes the properties of polygons and their components.

B. Analyzes the properties of circles and the lines that intersect them.

C. Uses geometric patterns and properties (e.g., similarity, congruence) to make generalizations about two- and three-dimensional figures and shapes (e.g., relationships of sides, angles).

D. Computes the perimeter, area and volume of figures and shapes created by subdividing and combining other figures and shapes (e.g., arc length, area of sectors).

E. Analyzes cross-sections and nets of three-dimensional shapes.

F. Uses top, front, side and corner views of three-dimensional shapes to create complete representations and solve problems.

G. Applies properties of two- and three-dimensional shapes to solve problems across the curriculum and in everyday life.

**Competency 014—The teacher understands coordinate, transformational and vector geometry and their connections.**

The beginning teacher:

A. Identifies transformations (i.e., reflections, translations, glide-reflections, rotations, dilations) and explores their properties.

B. Uses the properties of transformations and their compositions to solve problems.

C. Uses transformations to explore and describe reflectional, rotational and translational symmetry.

D. Applies transformations in the coordinate plane.

E. Applies concepts and properties of slope, midpoint, parallelism, perpendicularity and distance to explore properties of geometric figures and solve problems in the coordinate plane.

F. Uses coordinate geometry to derive and explore the equations, properties and applications of conic sections (i.e., lines, circles, hyperbolas, ellipses, parabolas).

G. Relates geometry and algebra by representing transformations as matrices and uses this relationship to solve problems.

H. Explores the relationship between geometric and algebraic representations of vectors and uses this relationship to solve problems.

## Domain IV—Probability and Statistics

**Competency 015—The teacher understands how to use appropriate graphical and numerical techniques to explore data, characterize patterns and describe departures from patterns.**

The beginning teacher:

A. Selects and uses an appropriate measurement scale (i.e., nominal, ordinal, interval, ratio) to answer research questions and analyze data.

B. Organizes, displays and interprets data in a variety of formats (e.g., tables, frequency distributions, scatter plots, stem-and-leaf plots, box-and-whisker plots, histograms, pie charts).

C. Applies concepts of center, spread, shape and skewness to describe a data distribution.

D. Understands measures of central tendency (i.e., mean, median, mode) and dispersion (i.e., range, interquartile range, variance, standard deviation).

E. Applies linear transformations (i.e., translating, stretching, shrinking) to convert data and describes the effect of linear transformations on measures of central tendency and dispersion.

F. Analyzes connections among concepts of center and spread, data clusters and gaps, data outliers and measures of central tendency and dispersion.

G. Supports arguments, makes predictions and draws conclusions using summary statistics and graphs to analyze and interpret one-variable data.

**Competency 016—The teacher understands concepts and applications of probability.**

The beginning teacher:

A. Understands how to explore concepts of probability through sampling, experiments and simulations and generates and uses probability models to represent situations.

B. Uses the concepts and principles of probability to describe the outcomes of simple and compound events.

C. Determines probabilities by constructing sample spaces to model situations.

D. Solves a variety of probability problems using combinations and permutations.

E. Solves a variety of probability problems using ratios of areas of geometric regions.

F. Calculates probabilities using the axioms of probability and related theorems and concepts such as the addition rule, multiplication rule, conditional probability and independence.

G. Understands expected value, variance and standard deviation of probability distributions (e.g., binomial, geometric, uniform, normal).

H. Applies concepts and properties of discrete and continuous random variables to model and solve a variety of problems involving probability and probability distributions (e.g., binomial, geometric, uniform, normal).

**Competency 017—The teacher understands the relationships among probability theory, sampling and statistical inference and how statistical inference is used in making and evaluating predictions.**

The beginning teacher:

A. Applies knowledge of designing, conducting, analyzing and interpreting statistical experiments to investigate real-world problems.

B. Analyzes and interprets statistical information (e.g., the results of polls and surveys) and recognizes misleading as well as valid uses of statistics.

C. Understands random samples and sample statistics (e.g., the relationship between sample size and confidence intervals, biased or unbiased estimators).

D. Makes inferences about a population using binomial, normal and geometric distributions.

E. Describes and analyzes bivariate data using various techniques (e.g., scatterplots, regression lines, outliers, residual analysis, correlation coefficients).

F. Understands how to transform nonlinear data into linear form to apply linear regression techniques to develop exponential, logarithmic and power regression models.

G. Uses the law of large numbers and the central limit theorem in the process of statistical inference.

H. Estimates parameters (e.g., population mean and variance) using point estimators (e.g., sample mean and variance).

I. Understands principles of hypotheses testing.

## Domain V—Mathematical Processes and Perspectives

**Competency 018—The teacher understands mathematical reasoning and problem solving.**

The beginning teacher:

A. Understands the nature of proof, including indirect proof, in mathematics.

B. Applies correct mathematical reasoning to derive valid conclusions from a set of premises.

C. Uses inductive reasoning to make conjectures and uses deductive methods to evaluate the validity of conjectures.

D. Uses formal and informal reasoning to justify mathematical ideas.

E. Understands the problem-solving process (i.e., recognizing that a mathematical problem can be solved in a variety of ways, selecting an appropriate strategy, evaluating the reasonableness of a solution).

F. Evaluates how well a mathematical model represents a real-world situation.

**Competency 019—The teacher understands mathematical connections both within and outside of mathematics and how to communicate mathematical ideas and concepts.**

The beginning teacher:

A. Recognizes and uses multiple representations of a mathematical concept (e.g., a point and its coordinates, the area of a circle as a quadratic function of the radius, probability as the ratio of two areas, area of a plane region as a definite integral).

B. Understands how mathematics is used to model and solve problems in other disciplines (e.g., art, music, science, social science, business).

C. Translates mathematical ideas between verbal and symbolic forms.

D. Communicates mathematical ideas using a variety of representations (e.g., numeric, verbal, graphical, pictorial, symbolic, concrete).

E. Understands the use of visual media, such as graphs, tables, diagrams and animations, to communicate mathematical information.

F. Uses appropriate mathematical terminology to express mathematical ideas.

## Domain VI—Mathematical Learning, Instruction and Assessment

**Competency 020—The teacher understands how children learn mathematics and plans, organizes and implements instruction using knowledge of students, subject matter and statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).**

The beginning teacher:

A. Applies research-based theories of learning mathematics to plan appropriate instructional activities for all students.

B. Understands how students differ in their approaches to learning mathematics.

C. Uses students' prior mathematical knowledge to build conceptual links to new knowledge and plans instruction that builds on students' strengths and addresses students' needs.

D. Understands how learning may be enhanced through the use of manipulatives, technology and other tools (e.g., stop watches, rulers).

E. Understands how to provide instruction along a continuum from concrete to abstract.

F. Understands a variety of instructional strategies and tasks that promote students' abilities to do the mathematics described in the TEKS.

G. Understands how to create a learning environment that provides all students, including English-language learners, with opportunities to develop and improve mathematical skills and procedures.

H. Understands a variety of questioning strategies to encourage mathematical discourse and to help students analyze and evaluate their mathematical thinking.

I. Understands how to relate mathematics to students' lives and to a variety of careers and professions.

**Competency 021—The teacher understands assessment and uses a variety of formal and informal assessment techniques to monitor and guide mathematics instruction and to evaluate student progress.**

The beginning teacher:

A. Understands the purpose, characteristics and uses of various assessments in mathematics, including formative and summative assessments.

B. Understands how to select and develop assessments that are consistent with what is taught and how it is taught.

C. Understands how to develop a variety of assessments and scoring procedures consisting of worthwhile tasks that assess mathematical understanding, common misconceptions and error patterns.

D. Understands the relationship between assessment and instruction and knows how to evaluate assessment results to design, monitor and modify instruction to improve mathematical learning for all students, including English-language learners.

This reference material will also be available to you during the exam. To access it, click on the

Reference Materials icon located in the lower-left corner of the screen.

## *Definitions and Formulas*

### CALCULUS

First Derivative: $f'(x) = \frac{dy}{dx}$

Second Derivative: $f''(x) = \frac{d^2y}{dx^2}$

### PROBABILITY

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$

### ALGEBRA

i $\quad i^2 = -1$

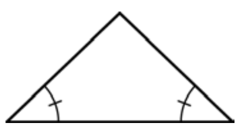
$\boldsymbol{A}^{-1}$ $\quad$ inverse of matrix A

$\boldsymbol{A} = \boldsymbol{P}\left(\boldsymbol{1} + \frac{\boldsymbol{r}}{\boldsymbol{n}}\right)^{\boldsymbol{nt}}$ $\quad$ Compound interest,
where A is the final value
P is the principal
r is the interest rate
t is the term
n is the number of divisions within the term

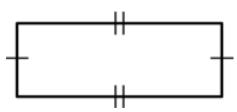
$[\boldsymbol{x}] = \boldsymbol{n}$ $\quad$ Greatest integer function,
where n is the integer such that $n \le x < n + 1$

### GEOMETRY

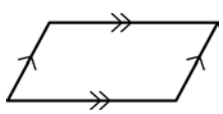
**Congruent Angles**



**Congruent Sides**



**Parallel Sides**



**Circumference of a Circle**

$C = 2\pi r$

### VOLUME

Cylinder: (area of base) $\times$ height

Cone: $\frac{1}{3}$(area of base) $\times$ height

Sphere: $\frac{4}{3}\pi r^3$

Prism: (area of base) $\times$ height

### AREA

Triangle: $\frac{1}{2}(\text{base} \times \text{height})$

Rhombus: $\frac{1}{2}(\text{diagonal}_1 \times \text{diagonal}_2)$

Trapezoid: $\frac{1}{2}\text{height}(\text{base}_1 + \text{base}_2)$

Sphere: $4\pi r^2$

Circle: $\pi r^2$

Lateral surface area of cylinder: $2\pi rh$

### TRIGONOMETRY

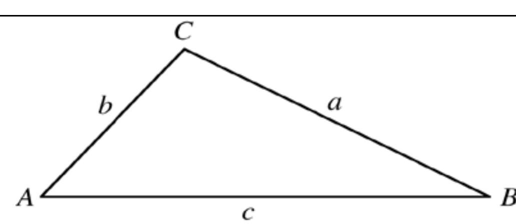
Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Law of Cosines:

$c^2 = a^2 + b^2 - 2ab\cos C$

$b^2 = a^2 + c^2 - 2ac\cos B$

$a^2 = b^2 + c^2 - 2bc\cos A$




## End of Definitions and Formulas

# Section 4: Sample Selected-Response Questions Mathematics 7–12 (235)

This section presents some sample exam questions for you to review as part of your preparation for the exam. To demonstrate how each competency may be assessed, sample questions are accompanied by the competency that they measure. While studying, you may wish to read the competency before and after you consider each sample question. Please note that the competency statements do not appear on the actual exam.

For each sample exam question, there is a correct answer and a rationale for each answer option. The sample questions are included to illustrate the formats and types of questions you will see on the exam; however, your performance on the sample questions should not be viewed as a predictor of your performance on the actual exam.

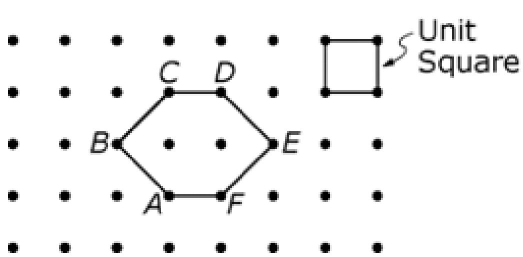
The following reference materials will be available to you during the exam:

- Definitions and Formulas (see page 14)

## Domain I—Number Concepts

**Competency 001—The teacher understands the real number system and its structure, operations, algorithms and representations.**

1. **Use the figure below to answer the question that follows.**




The figure above represents a geoboard, and each unit square has area 1. Which of the following quantities associated with hexagon *ABCDEF* is an integer?

A. The length of $\overline{BD}$

B. The area of triangle *BCE*

C. The area of hexagon *ABCDEF*

D. The distance from *B* to the midpoint of $\overline{BE}$

**Answer** _____

2. *S* is the set of all positive integers that can be written in the form $2^n \cdot 3^m$, where *n* and *m* are positive integers. If *a* and *b* are two numbers in *S*, which of the following must also be in *S*?

A. $a + b$

B. $\sqrt{ab}$

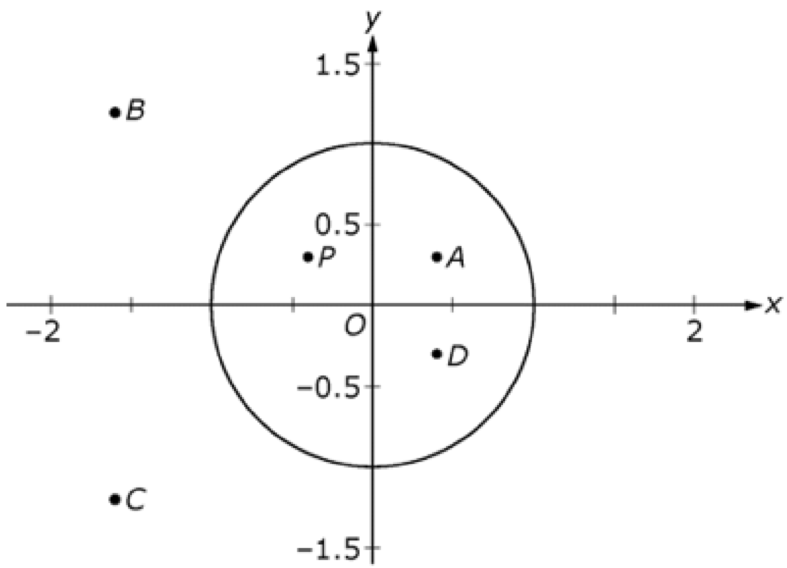
C. $\frac{a}{b}$

D. $ab$

**Answer _____**

**Competency 002—The teacher understands the complex number system and its structure, operations, algorithms and representations.**

**Use the figure below to answer the question that follows.**




3. The figure above shows a unit circle in the complex plane. Which of the following points could represent the multiplicative inverse of the complex number represented by point *P*, which has coordinates (–0.4, 0.3)?

A. *A*

B. *B*

C. *C*

D. *D*

**Answer _____**

**Competency 003—The teacher understands number theory concepts and principles and uses numbers to model and solve problems in a variety of situations.**

4. Olivia traveled 25 miles in 30 minutes, and then she traveled for an additional 20 minutes. If her average speed for the entire trip was 36 miles per hour (mph), what was her average speed for the final 20 minutes of the trip?

A. 15 mph

B. 20 mph

C. 25 mph

D. 30 mph

**Answer** _____

5. A researcher measured the length of an object to be $k$ centimeters, where $k < 0.00001$. The researcher expressed the value of $k$ in the form $a \times 10^b$, where $a$ is a real number and $b$ is an integer. Which of the following could be true about $a$ and $b$ in this situation?

A. $-1 \leq a < 0$ and $b < -1$

B. $1 \leq a < 10$ and $b < -1$

C. $1 \leq a < 10$ and $b > 1$

D. $\frac{1}{2} \leq a < 1$ and $b > 1$

**Answer** _____

## Domain II—Patterns and Algebra

**Competency 004—The teacher uses patterns to model and solve problems and formulate conjectures.**

6. If $\{a_n\}_{n=1}^{\infty}$ is a sequence such that $a_1 = 1$, $a_2 = 3$, and $a_{n+3} = \frac{a_{n+1}}{a_{n+2}}$ for all integers $n \geq 0$, what is the value of $a_4$?

A. 9

B. 7

C. 1

D. $\frac{1}{3}$

**Answer** _____

7. A certain finite sequence of consecutive integers begins with –13. If the sum of all the terms of the sequence is 45, how many terms are there in the sequence?

A. 27

B. 28

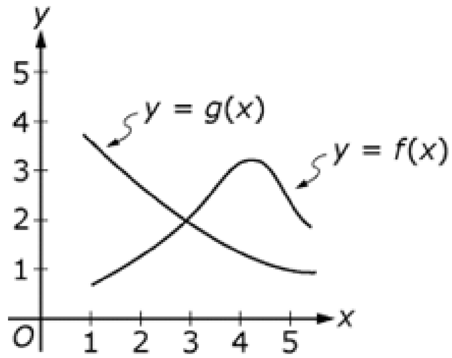
C. 29

D. 30

**Answer** _____

**Competency 005—The teacher understands attributes of functions, relations and their graphs.**

8. **Use the graph below to answer the question that follows.**




The graphs of the functions *f* and *g* are shown in the *xy*-plane above. For which of the following values of *x* is the value of *g*(*x*) closest to the value of *f*(2)?

A. 1

B. 2

C. 3

D. 4

**Answer _____**

9. Let *f* be the function defined by $f(x) = -x + \frac{1}{x}$ for all $x \neq 0$. Which of the following must be true?

A. $f(-x) = -f(x)$

B. $f(-x) = f(x)$

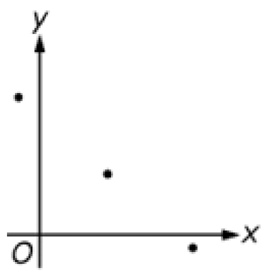
C. $f(\frac{1}{x}) = f(x)$

D. $f(\frac{1}{x}) = -\frac{1}{f(x)}$

**Answer _____**

**Competency 006—The teacher understands linear and quadratic functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.**

10. **Use the graph below to answer the question that follows.**




A nonvertical line in the *xy*-plane can be represented by an equation of the form $y = mx + b$, where $m$ and $b$ are constants. If line $\ell$ contains the three points shown, which of the following statements about $m$ and $b$ is true for line $\ell$ ?

A. $m > 0$ and $b > 0$

B. $m > 0$ and $b < 0$

C. $m < 0$ and $b > 0$

D. $m < 0$ and $b < 0$

**Answer** _____

11. Let $f$ be the function defined for all real numbers $x$ by $f(x) = (x - a)^2 + b$, where $a$ and $b$ are constants such that $0 < a < b$. The function $f$ is one-to-one on which of the following intervals?

A. $0 < x < b$

B. $0 < x < 2a$

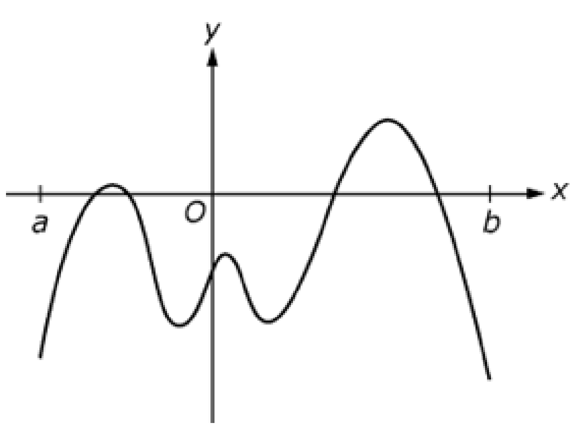
C. $-b < x < b$

D. $-b < x < a$

**Answer** _____

**Competency 007—The teacher understands polynomial, rational, radical, absolute value and piecewise functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.**

12. **Use the graph below to answer the question that follows.**



The *xy*-plane above shows the graph of $y = f(x)$ on the closed interval $[a,b]$, where $f$ is a polynomial with real coefficients. The function $f$ is strictly increasing for all $x < a$ and is strictly decreasing for all $x > b$. Which of the following statements about $f$ is true?

A. $f$ has 6 real zeros and degree at least 6.

B. $f$ has 4 real zeros and degree at least 6.

C. $f$ has 4 real zeros and degree at most 5.

D. $f$ has 4 real zeros and degree at most 4.

**Answer** _____

13. **Use the equation below to answer the question that follows.**

$$y = x + 3 - \frac{1}{x - 2}$$

Which of the following is an equation of one of the asymptotes of the graph, in the *xy*-plane, of the equation above?

A. $x = -3$

B. $x = 1$

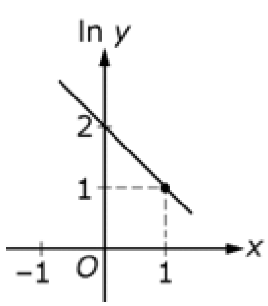
C. $y = x - 2$

D. $y = x + 3$

**Answer** _____

**Competency 008—The teacher understands exponential and logarithmic functions, analyses their algebraic and graphical properties and uses them to model and solve problems.**

14. **Use the figure below to answer the question that follows.**




If $x$ and $y$ are related by the line shown above, which of the following equations gives $y$ in terms of $x$?

A. $y = e^x + 2$

B. $y = 2e^x$

C. $y = e^{x+2}$

D. $y = e^{2-x}$

**Answer** _____

**Use the formula and information below to answer the question that follows.**

In a bank account in which interest is compounded continuously, the amount $A$ in the account is given by $A = Pe^{rt}$, where $P$ is the initial deposit, $r$ is the annual interest rate, and $t$ is the time in years.

15. Felicia opens a bank account that pays interest compounded continuously at the annual rate of 2.5%. Her initial deposit is $2000, and there will be no other transactions until the amount in her account is $2500. Based on the formula given above, how many years, to the nearest whole number of years, will it take until she has $2500 in the account?

A. 9

B. 10

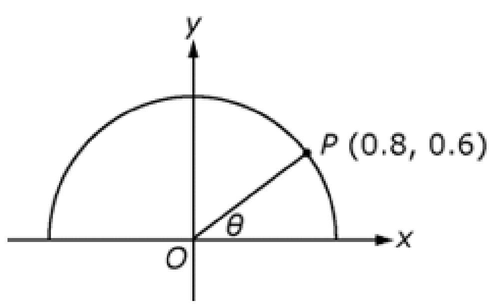
C. 11

D. 12

**Answer _____**

**Competency 009—The teacher understands trigonometric and circular functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.**

**Use the figure below to answer the question that follows.**




16. In the $xy$-plane above, point $P$ lies on the semicircle with center $O$. What is the value of $\theta$?

A. $\cos^{-1} 0.6$

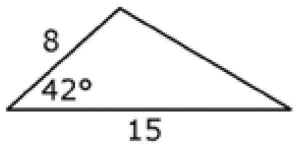
B. $\sin^{-1} 0.75$

C. $\sin^{-1} 0.8$

D. $\tan^{-1} 0.75$

**Answer _____**

17. **Use the figure below to answer the question that follows.**




What is the area of the triangle above?

A. 60 sin 42°

B. 60 cos 42°

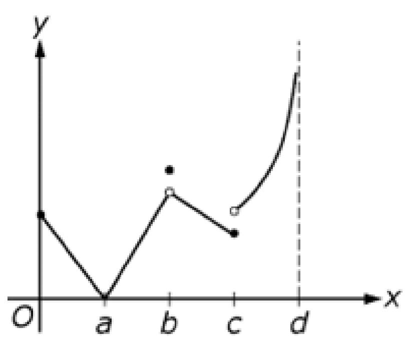
C. 120 sin 42°

D. 120 tan 42°

**Answer _____**

**Competency 010—The teacher understands and solves problems using differential and integral calculus.**

18. **Use the graph below to answer the question that follows.**




The graph of the function *f* on the interval $0 \le x < d$ is shown above, where $\lim_{x \to d^-} f(x) = +\infty$. For which of the following values of *x* does *f* have a removable discontinuity?

A. *a*

B. *b*

C. *c*

D. *d*

**Answer _____**

# Domain III—Geometry and Measurement

**Competency 011—The teacher understands measurement as a process.**

**Use the information below to answer the question that follows.**

The surface area of a roof is measured in squares of shingles. Each square of shingles covers 100 square feet. However, shingles are sold in bundles and priced per bundle. It takes 3 bundles of shingles to make a square of shingles.

19. A certain roof consists of 2 rectangular sides, each having dimensions 15 feet by 60 feet. Based on the information above, if shingles cost $28.99 per bundle, which of the following represents the total cost of the shingles for the roof?

A. $\frac{(3)(2)(15)(60)(\$28.99)}{100}$

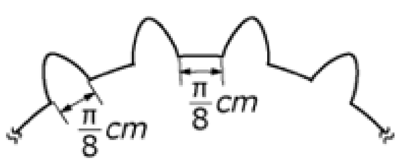
B. $\frac{(2)(15)(60)(\$28.99)(100)}{3}$

C. $\frac{(2)(15)(60)(\$28.99)}{(3)(100)}$

D. $\frac{(3)(\$28.99)(100)}{(2)(15)(60)}$

**Answer _____**

20. **Use the figure below to answer the question that follows.**




The figure shows a portion of a gear that has cogs evenly spaced around the circumference of a wheel. Each cog is $\frac{\pi}{8}$ centimeters wide, and there is a space of $\frac{\pi}{8}$ centimeters between consecutive cogs. If the diameter of the wheel is 9 centimeters, how many cogs are on the wheel?

A. 12

B. 18

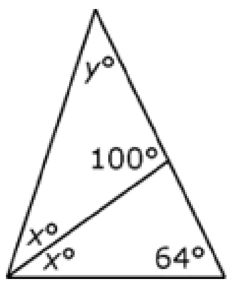
C. 24

D. 36

**Answer _____**

Competency 012—The teacher understands geometries, in particular Euclidian geometry, as axiomatic systems.

**Use the figure below to answer the question that follows.**




21. What is the value of *y* in the triangle above?

A. 36

B. 40

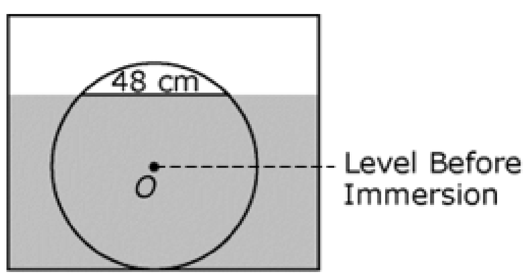
C. 44

D. 48

**Answer _____**

Competency 013—The teacher understands the results, uses and applications of Euclidian geometry.

**Use the figure below to answer the question that follows.**




Note: Figure not drawn to scale.

22. A wheel with center *O* and radius 25 cm is immersed in a vat of cleaning solution, as shown in the figure above. The chord of length 48 cm indicates the solution level after the wheel was immersed. The dashed line indicates the solution level before the wheel was immersed. What is the level of the solution in the vat after the wheel has been immersed?

A. 32 cm

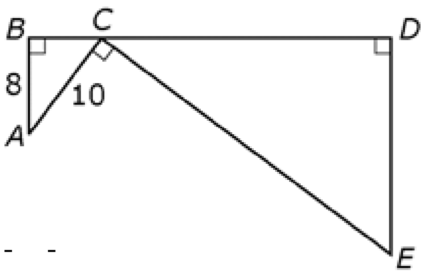
B. 33 cm

C. 35 cm

D. 37 cm

**Answer _____**

**Use the figure below to answer the question that follows.**




23. In the figure above, $C$ is a point on $\overline{BD}$. Triangles $ABC$ and $CDE$ are right triangles, and $\overline{AC} \perp \overline{CE}$. If the length of $\overline{BD}$ is 30, what is the length of $\overline{DE}$?

A. 18

B. 20

C. 24

D. 32

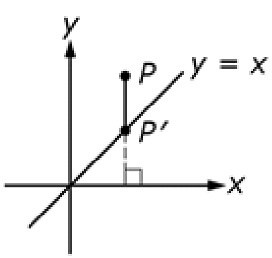
**Answer _____**

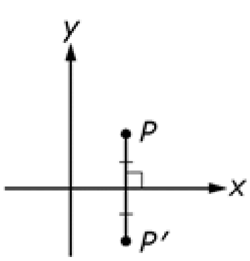
**Competency 014—The teacher understands coordinate, transformational and vector geometry and their connections.**

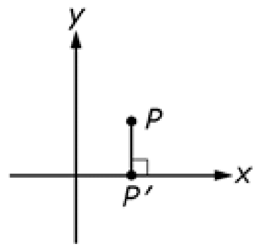
**Use the matrix equation below to answer the question that follows.**

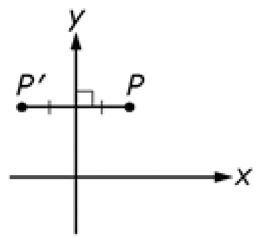
$$\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$$

24. The matrix equation above defines a transformation of the *xy*-plane. Which of the following shows a point $P$ and its image $P'$ under this transformation?

A. 


B. 

C. 


D. 


**Answer _____**

# Domain IV—Probability and Statistics

**Competency 015—The teacher understands how to use appropriate graphical and numerical techniques to explore data, characterize patterns and describe departures from patterns.**

**Use the definition below to answer the question that follows.**

For a set of data, a data point is an outlier if it is more than 1.5 times the interquartile range of the data set either above the third quartile or below the first quartile.

The bulb life, in hours, for 27 lightbulbs of the same brand is recorded below.

| | | | | | |
|---|---|---|---|---|---|
| 275 | 400 | 431 | 465 | 480 | 495 |
| 350 | 400 | 436 | 465 | 481 | 595 |
| 360 | 420 | 450 | 470 | 483 | |
| 380 | 425 | 452 | 473 | 490 | |
| 395 | 428 | 460 | 474 | 492 | |

25. Based on the definition above, which of the numbers 275 and 595 is an outlier?

A. Neither 275 nor 595

B. 275 only

C. 595 only

D. Both 275 and 595

**Answer _____**

**Competency 016—The teacher understands concepts and applications of probability.**

26. A computer company employs over 4000 employees, of whom 45% are women. If a focus group of 20 randomly selected employees is to be formed, what is the expected number of men in the focus group?

A. 8

B. 9

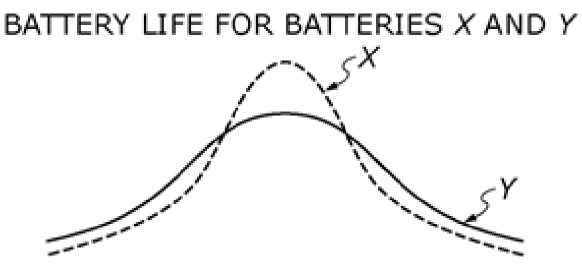
C. 11

D. 13

**Answer _____**

**Competency 017—The teacher understands the relationships among probability theory, sampling and statistical inference and how statistical inference is used in making and evaluating predictions.**

**Use the graph below to answer the question that follows.**




27. The battery life, in years, for each of two brands of car batteries, *X* and *Y*, is approximately normally distributed, as shown above. Which of the following statements about the mean and standard deviation of battery life for the two distributions is true?

A. The mean battery life for *X* is less than the mean battery life for *Y.*

B. The mean battery life for *X* is greater than the mean battery life for *Y.*

C. The standard deviation of battery life for *X* is less than the standard deviation of battery life for *Y.*

D. The standard deviation of battery life for *X* is greater than the standard deviation of battery life for *Y.*

**Answer _____**

28. To evaluate a new medication that was developed to reduce the occurrence of headaches, a randomized controlled experiment is conducted. One-third of the patients are given the new medication, one-third are given a placebo, and one-third are given nothing. Which of the following is the best example of the placebo effect for this study?

A. People taking the placebo report more headaches than people taking the new medication.

B. People taking the placebo report fewer headaches than people taking the new medication.

C. People taking the placebo report more headaches than people taking nothing.

D. People taking the placebo report fewer headaches than people taking nothing.

**Answer _____**

# Domain V—Mathematical Processes and Perspectives

**Competency 018—The teacher understands mathematical reasoning and problem solving.**

**Use the statement below to answer the question that follows.**

If $x^2$ is even, then $x$ is even.

29. A student is trying to prove that the statement above is true for all integers $x$ by proving its contrapositive. Which of the following procedures should the student follow in order to use this method of proof?

A. Assume that $x^2$ is even, and then deduce that $x$ is even

B. Assume that $x^2$ is not even, and then deduce that $x$ is not even

C. Assume that $x^2$ is even, and then deduce that $x$ is not even

D. Assume that $x$ is not even, and then deduce that $x^2$ is not even

**Answer _____**

**Competency 019—The teacher understands mathematical connections both within and outside of mathematics and how to communicate mathematical ideas and concepts.**

**Use the problem below to answer the question that follows.**

Working together at their constant rates, hoses *A* and *B* can fill an empty pool in 10 hours. Working alone, it takes hose *B* twice as many hours as hose *A* to fill the pool. How many hours would it take hose *A*, working alone at its constant rate, to fill the pool?

30. In the problem above, if $x$ represents the number of hours it takes hose *A* to fill the pool working alone, which of the following equations correctly models the situation?

A. $\frac{1}{x} + \frac{1}{2x} = \frac{1}{10}$

B. $\frac{1}{x} + \frac{2}{x} = \frac{1}{10}$

C. $\frac{1}{x} + \frac{1}{2x} = 10$

D. $x + 2x = 10$

**Answer _____**

# Domain VI—Mathematical Learning, Instruction and Assessment

**Competency 020—The teacher understands how children learn mathematics and plans, organizes and implements instruction using knowledge of students, subject matter and statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).**

31. Of the following activities involving the quadratic expression $ax^2 + bx + c$, which best exemplifies inquiry-based learning?

A. Students predict how the graph of $y = ax^2 + bx + c$ will be affected by changing the value of $a$, and check their predictions using a graphing calculator.

B. Students solve an equation of the form $ax^2 + bx + c = 0$ by graphing the equation on a graphing calculator.

C. Students derive the quadratic formula by completing the square on the left side of the equation $ax^2 + bx + c = 0$.

D. Students use a function of the form $f(x) = ax^2 + bx + c$ to model a problem involving falling bodies.

**Answer** _____

**Competency 021—The teacher understands assessment and uses a variety of formal and informal assessment techniques to monitor and guide mathematics instruction and to evaluate student progress.**

32. If a student mistakenly states that $-\frac{1}{2}(-\frac{2}{3}x + \frac{1}{2}) = \frac{1}{3}x + \frac{1}{2}$, it is most likely that the mistake results from a misunderstanding of which of the following?

A. Multiplication of fractions

B. Arithmetic of negative numbers

C. Associative property of multiplication

D. Distributive property of multiplication over addition

**Answer** _____

# Section 4: Sample Selected-Response Answers and Rationales Mathematics 7–12 (235)

This section presents some sample exam questions for you to review as part of your preparation for the exam. To demonstrate how each competency may be assessed, sample questions are accompanied by the competency that they measure. While studying, you may wish to read the competency before and after you consider each sample question. Please note that the competency statements do not appear on the actual exam.

For each sample exam question, there is a correct answer and a rationale for each answer option. The sample questions are included to illustrate the formats and types of questions you will see on the exam; however, your performance on the sample questions should not be viewed as a predictor of your performance on the actual exam.

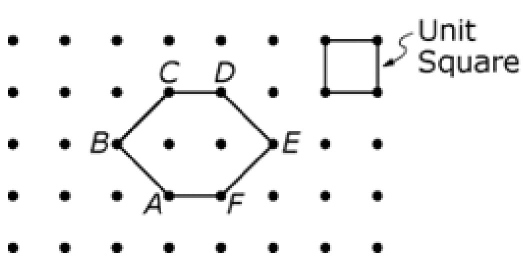
The following reference materials will be available to you during the exam:

- Definitions and Formulas (see page 14)

## Domain I—Number Concepts

**Competency 001—The teacher understands the real number system and its structure, operations, algorithms and representations.**

1. **Use the figure below to answer the question that follows.**




The figure above represents a geoboard, and each unit square has area 1. Which of the following quantities associated with hexagon *ABCDEF* is an integer?

A. The length of $\overline{BD}$

B. The area of triangle *BCE*

C. The area of hexagon *ABCDEF*

D. The distance from *B* to the midpoint of $\overline{BE}$

**Answer**

**Option C is correct** because the area of hexagon *ABCDEF* is equal to 4 square units, and 4 is an integer. Each unit square has area 1, and the hexagon is composed of 2 full squares and 4 half-squares, for a total area of 2(1) + 4(0.5) = 2 + 2 = 4. **Option A is incorrect** because, by the Pythagorean theorem, the length of $\overline{BD}$ is $\sqrt{2^2 + 1^2}$ =

$\sqrt{5}$, which is not an integer. **Option B is incorrect** because the area of triangle *BCE* is $\frac{1}{2}bh = \frac{1}{2}(3)(1) = 1.5$, which is not an integer. **Option D is incorrect** because the distance from *B* to the midpoint of $\overline{BE}$ is $\frac{3}{2} = 1.5$, which is not an integer.

2. *S* is the set of all positive integers that can be written in the form $2^n \cdot 3^m$, where *n* and *m* are positive integers. If *a* and *b* are two numbers in *S*, which of the following must also be in *S*?

A. $a + b$

B. $\sqrt{ab}$

C. $\frac{a}{b}$

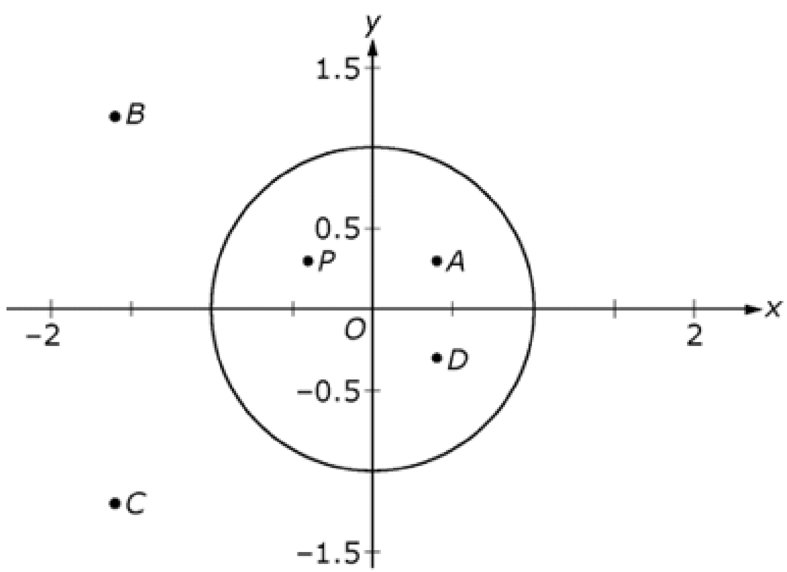
D. $ab$

**Answer**

**Option D is correct** because if *a* and *b* are two numbers in *S*, then $a = 2^j \cdot 3^k$ and $b = 2^s \cdot 3^t$, where *j*, *k*, *s*, and *t* are positive integers. So $ab = (2^j \cdot 3^k)(2^s \cdot 3^t) = 2^{j+s} \cdot 3^{k+t}$, and $j + s$ and $k + t$ are both positive integers; thus *ab* is in *S*. **Option A is incorrect** because, for example, if $a = 18$ and $b = 12$, then $a + b = 30$, which is not in *S*. **Options B and C are incorrect** because, for example, if $a = 6$ and $b = 12$, then $\sqrt{ab} = \sqrt{72} = 6\sqrt{2}$ and $\frac{a}{b} = \frac{6}{12} = \frac{1}{2}$, neither of which is in *S*.

**Competency 002—The teacher understands the complex number system and its structure, operations, algorithms and representations.**

**Use the figure below to answer the question that follows.**



3. The figure above shows a unit circle in the complex plane. Which of the following points could represent the multiplicative inverse of the complex number represented by point *P*, which has coordinates (–0.4, 0.3)?

A. *A*

B. *B*

C. *C*

D. *D*

**Answer**

**Option C is correct** because the multiplicative inverse of a complex number $a + bi$ is $\frac{a}{a^2 + b^2} - \frac{b}{a^2 + b^2}i$. Point *P* represents the complex number –0.4 + 0.3*i*, which has multiplicative inverse $\frac{-0.4}{(-0.4)^2 + (0.3)^2} - \frac{0.3}{(-0.4)^2 + (0.3)^2}i$, or –1.6 − 1.2*i*. This number is represented by the point with coordinates (–1.6,–1.2), which can only be point *C*. **Options A and D are incorrect** because the multiplicative inverse cannot be obtained by reflecting *P* across the *y*-axis and the origin, respectively. **Option B is incorrect** because the inverse does not lie on the ray *OP*.

**Competency 003—The teacher understands number theory concepts and principles and uses numbers to model and solve problems in a variety of situations.**

4. Olivia traveled 25 miles in 30 minutes, and then she traveled for an additional 20 minutes. If her average speed for the entire trip was 36 miles per hour (mph), what was her average speed for the final 20 minutes of the trip?

A. 15 mph

B. 20 mph

C. 25 mph

D. 30 mph

**Answer**

**Option A is correct.** Olivia traveled at an average speed of 36 mph for 50 minutes, or $\frac{5}{6}$ of an hour. This gives a total distance of $36(\frac{5}{6}) = 30$ miles. She traveled 25 miles in the first 30 minutes, leaving only 5 miles in the last 20 minutes. A rate of 5 miles in 20 minutes is equivalent to a rate of 15 miles in an hour. **Option B is incorrect** because if Olivia had traveled at a rate of 20 mph for the last 20 minutes, her average speed for the entire trip would have been $\frac{25 + \frac{20}{3}}{\frac{5}{6}} = 38$ mph. **Option C is incorrect** because if Olivia had traveled at a rate of 25 mph for the last 20 minutes, her average speed for the entire trip would have been $\frac{25 + \frac{25}{3}}{\frac{5}{6}} = 40$ mph. **Option D is incorrect**

because if Olivia had traveled at a rate of 30 mph for the last 20 minutes, her average speed for the entire trip would have been $\dfrac{25 + \dfrac{30}{3}}{\dfrac{5}{6}} = 42$ mph.

5. A researcher measured the length of an object to be $k$ centimeters, where $k < 0.00001$. The researcher expressed the value of $k$ in the form $a \times 10^b$, where $a$ is a real number and $b$ is an integer. Which of the following could be true about $a$ and $b$ in this situation?

A. $-1 \le a < 0$ and $b < -1$

B. $1 \le a < 10$ and $b < -1$

C. $1 \le a < 10$ and $b > 1$

D. $\frac{1}{2} \le a < 1$ and $b > 1$

**Answer**

**Option B is correct** because $0.00001 = 10^{-5} = 10 \times 10^{-6}$, $k$ can be of the form $a \times 10^b$ for $1 \le a < 10$ and $b < -1$. For example, if $k = 0.000002$, then $a = 2$ and $b = -6$. **Option A is incorrect** because if $a$ is negative, then the value of $a \times 10^b$ will also be negative and thus cannot represent a distance. **Option C is incorrect** because if $1 \le a < 10$ and $b > 1$, then $a \times 10^b > 10$. **Option D is incorrect** because if $\frac{1}{2} \le a < 1$ and $b > 1$, then $a \times 10^b > 5$.

## Domain II—Patterns and Algebra

**Competency 004—The teacher uses patterns to model and solve problems and formulate conjectures.**

6. If $\{a_n\}_{n=1}^{\infty}$ is a sequence such that $a_1 = 1$, $a_2 = 3$, and $a_{n+3} = \dfrac{a_{n+1}}{a_{n+2}}$ for all integers $n \ge 0$, what is the value of $a_4$?

A. 9

B. 7

C. 1

D. $\frac{1}{3}$

**Answer**

**Option A is correct** because by the given formula, $a_3 = \dfrac{a_1}{a_2} = \dfrac{1}{3}$ and $a_4 = \dfrac{a_2}{a_3} = \dfrac{3}{(\frac{1}{3})} = 9$. **Option B is incorrect** because 7 is the result obtained by adding the two previous terms each time, instead of taking the quotient. **Option**

**C is incorrect** because 1 is the result obtained by using $a_{n+3} = \frac{a_{n+2}}{a_{n+1}}$. **Option D is incorrect** because $\frac{1}{3}$ is the value of $a_3$ instead of $a_4$.

7. A certain finite sequence of consecutive integers begins with –13. If the sum of all the terms of the sequence is 45, how many terms are there in the sequence?

A. 27

B. 28

C. 29

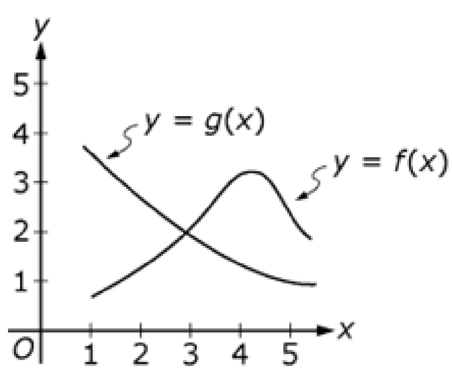
D. 30

**Answer**

**Option D is correct** because the terms of the sequence described are the consecutive integers starting at –13, the first 27 terms of the sequence, from –13 to 13, have a sum of 0. The next 3 terms, 14, 15 and 16, have a sum of 45, which is the given sum. So, there are 30 terms in the sequence. **Option A is incorrect** because the first 27 terms in the sequence have a sum of 0. **Option B is incorrect** because the first 28 terms in the sequence have a sum of 14. **Option C is incorrect** because the first 29 terms in the sequence have a sum of 29.

**Competency 005—The teacher understands attributes of functions, relations and their graphs.**

8. **Use the graph below to answer the question that follows.**




The graphs of the functions *f* and *g* are shown in the *xy*-plane above. For which of the following values of *x* is the value of *g*(*x*) closest to the value of *f*(2)?

A. 1

B. 2

C. 3

D. 4

**Answer**

**Option D is correct.** The value of $f(2)$ is a little greater than 1, and so is the value of $g(4)$. For the other options, the value of $g(x)$ is not as close to the value of $f(2)$ as is the value of $g(4)$. **Option A is incorrect** because $g(1)$ is greater than 3. **Option B is incorrect** because $g(2)$ is greater than 2. **Option C is incorrect** because $g(3)$ is about 2.

9. Let $f$ be the function defined by $f(x) = -x + \frac{1}{x}$ for all $x \neq 0$. Which of the following must be true?

A. $f(-x) = -f(x)$

B. $f(-x) = f(x)$

C. $f(\frac{1}{x}) = f(x)$

D. $f(\frac{1}{x}) = -\frac{1}{f(x)}$

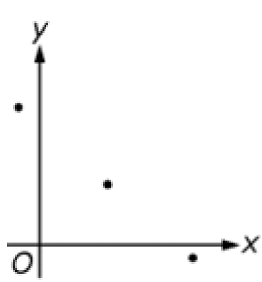
**Answer**

**Option A is correct** because if $f(x) = -x + \frac{1}{x}$, then $f(-x) = -(-x) + \frac{1}{-x} = x - \frac{1}{x}$ and $-f(x) = -(-x + \frac{1}{x}) = x - \frac{1}{x}$.

These two functions are equivalent. **Option B is incorrect** because $f(-x) = -(-x) + \frac{1}{-x} = x - \frac{1}{x}$ is not equivalent to $f(x)$. **Option C is incorrect** because $f(\frac{1}{x}) = -(\frac{1}{x}) + \frac{1}{\frac{1}{(\frac{1}{x})}}$ $= -\frac{1}{x} + x$ is not equivalent to $f(x)$. **Option D is incorrect** because $f(\frac{1}{x}) = -(\frac{1}{x}) + \frac{1}{\frac{1}{(\frac{1}{x})}} = -\frac{1}{x} + x$ is not equivalent to $-\frac{1}{f(x)} = -\frac{1}{-x + \frac{1}{x}} = -\frac{1}{\frac{-x^2}{x} + \frac{1}{x}} = \frac{-x}{-x^2 + 1} = \frac{x}{x^2 - 1}$.

**Competency 006—The teacher understands linear and quadratic functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.**

10. **Use the graph below to answer the question that follows.**




A nonvertical line in the $xy$-plane can be represented by an equation of the form $y = mx + b$, where $m$ and $b$ are constants. If line $\ell$ contains the three points shown, which of the following statements about $m$ and $b$ is true for line $\ell$ ?

A. $m > 0$ and $b > 0$

B. $m > 0$ and $b < 0$

C. $m < 0$ and $b > 0$

D. $m < 0$ and $b < 0$

**Answer**

**Option C is correct** because when a line is represented as an equation in the form $y = mx + b$, $m$ represents the slope and $b$ represents the $y$-intercept. The line containing the three points shown has a negative slope and a positive $y$-intercept, so $m < 0$ and $b > 0$. **Option A is incorrect** because if $m > 0$ and $b > 0$, the line would have a positive slope. **Option B is incorrect** because if $m > 0$ and $b < 0$, the line would have a positive slope and a negative $y$-intercept. **Option D is incorrect** because if $m < 0$ and $b < 0$, the line would have a negative $y$-intercept.

11. Let $f$ be the function defined for all real numbers $x$ by $f(x) = (x - a)^2 + b$, where $a$ and $b$ are constants such that $0 < a < b$. The function $f$ is one-to-one on which of the following intervals?

A. $0 < x < b$

B. $0 < x < 2a$

C. $-b < x < b$

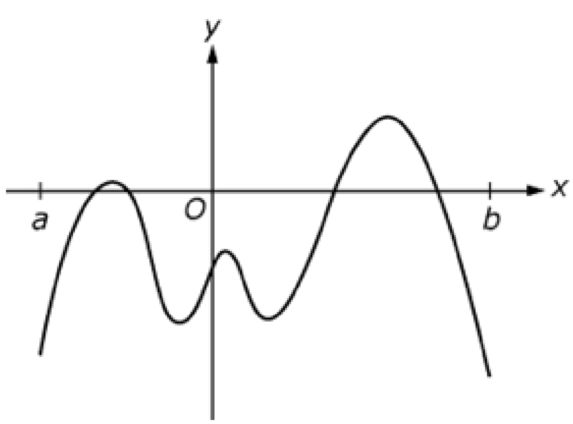
D. $-b < x < a$

**Answer**

**Option D is correct.** For a function to be one-to-one on an interval there must be exactly one $x$-value for each $y$-value. The graph of the function given is a parabola with vertex at $(a, b)$, where $0 < a < b$. On the interval $-b < x < a$, the parabola consists of points on the left side of the axis of symmetry. For these points, there is exactly one $x$-value for each $y$-value, so the function is one-to-one on this interval. **Options A, B, and C are incorrect** because the portion of the parabola on each of these intervals includes points on both sides of the axis of symmetry. This means that on each interval there are at least 2 points on the parabola with the same $y$-value but with different $x$-values.

**Competency 007—The teacher understands polynomial, rational, radical, absolute value and piecewise functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.**

12. **Use the graph below to answer the question that follows.**



The *xy*-plane above shows the graph of $y = f(x)$ on the closed interval $[a,b]$, where $f$ is a polynomial with real coefficients. The function $f$ is strictly increasing for all $x < a$ and is strictly decreasing for all $x > b$. Which of the following statements about $f$ is true?

A. $f$ has 6 real zeros and degree at least 6.

B. $f$ has 4 real zeros and degree at least 6.

C. $f$ has 4 real zeros and degree at most 5.

D. $f$ has 4 real zeros and degree at most 4.

**Answer**

**Option B is correct** because the fact that the graph intersects the $x$-axis in four places indicates that the function has 4 real zeros, and the fact that the graph has 5 local extrema indicates that the function has degree at least 6. **Option A is incorrect** because the graph cannot intersect the x-axis at more than the four places shown given the conditions on $f$ for $x < a$ and $x > b$. **Options C and D are incorrect** because the function must have degree at least 6.

13. **Use the equation below to answer the question that follows.**

$$y = x + 3 - \frac{1}{x - 2}$$

Which of the following is an equation of one of the asymptotes of the graph, in the *xy*-plane, of the equation above?

A. $x = -3$

B. $x = 1$

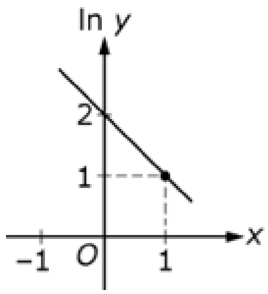
C. $y = x - 2$

D. $y = x + 3$

**Answer**

**Option D is correct** because as $x$ approaches $\infty$ or $-\infty$, the value of $\frac{1}{x-2}$ approaches 0; therefore, the value of $y = x + 3 + \frac{1}{x-2}$ approaches $y = x + 3$. **Options A and B are incorrect** because the only vertical asymptote of the graph occurs at $x = 2$. **Option C is incorrect** because $y = x - 2$ is not an asymptote of the graph.

14. **Use the figure below to answer the question that follows.**




If *x* and *y* are related by the line shown above, which of the following equations gives *y* in terms of *x*?

A. $y = e^x + 2$

B. $y = 2e^x$

C. $y = e^{x+2}$

D. $y = e^{2-x}$

**Answer**

**Option D is correct.** Based on the relationship shown in the graph, $\ln y = 2 - x$, so $y = e^{2-x}$. **Option A is incorrect** because if $y = e^x + 2$, then the relationship between $\ln y$ and $x$ would be $\ln y = \ln(e^x + 2)$, which is not represented on the graph. **Option B is incorrect** because if $y = 2e^x$, then the relationship between $\ln y$ and $x$ would be $\ln y = \ln(2e^x)$, which is not represented on the graph. **Option C is incorrect** because if $y = e^{x+2}$, then the relationship between $\ln y$ and $x$ would be $\ln y = x + 2$, which is not represented on the graph.

**Use the formula and information below to answer the question that follows.**

In a bank account in which interest is compounded continuously, the amount *A* in the account is given by $A = Pe^{rt}$, where *P* is the initial deposit, *r* is the annual interest rate, and *t* is the time in years.

15. Felicia opens a bank account that pays interest compounded continuously at the annual rate of 2.5%. Her initial deposit is $2000, and there will be no other transactions until the amount in her account is $2500. Based on the formula given above, how many years, to the nearest whole number of years, will it take until she has $2500 in the account?

A. 9

B. 10

C. 11

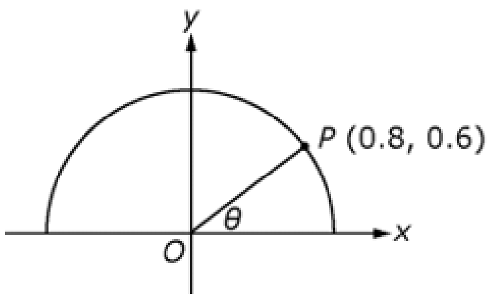
D. 12

**Answer**

**Option A is correct.** Based on the formula and information given, $2500 = 2000e^{0.025t}$. Solving for *t* yields $t = \frac{\ln 1.25}{0.025} \approx 8.9$, which to the nearest whole number is 9. **Options B, C, and D are incorrect** because they are greater than the number of years it takes for the value of the account to reach $2500.

**Competency 009—The teacher understands trigonometric and circular functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.**

**Use the figure below to answer the question that follows.**




16. In the *xy*-plane above, point *P* lies on the semicircle with center *O*. What is the value of $\theta$?

A. $\cos^{-1} 0.6$

B. $\sin^{-1} 0.75$

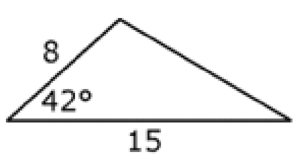
C. $\sin^{-1} 0.8$

D. $\tan^{-1} 0.75$

**Answer**

**Option D is correct** because when a vertical segment is drawn from point *P* to the *x*-axis, a right triangle is formed such that the vertical leg has length 0.6 and the horizontal leg has length 0.8. Thus, $\tan\theta = \frac{0.6}{0.8} = 0.75$, and $\theta = \tan^{-1} 0.75$. **Option A is incorrect** because in the right triangle described above, the length of the hypotenuse is $\sqrt{0.6^2 + 0.8^2} = 1$, so $\cos\theta = \frac{0.8}{1} = 0.8$ and $\theta = \cos^{-1} 0.8$. **Options B and C are incorrect** because in the right triangle described, $\sin\theta = \frac{0.6}{1} = 0.6$ and $\theta = \sin^{-1} 0.6$.

17. **Use the figure below to answer the question that follows.**




What is the area of the triangle above?

A. 60 sin 42°

B. 60 cos 42°

C. 120 sin 42°

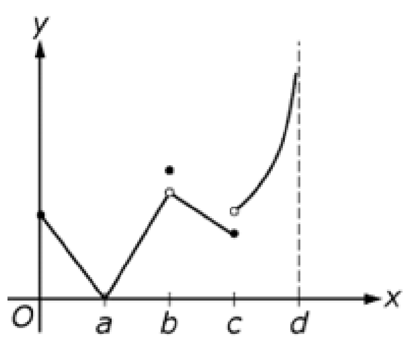
D. 120 tan 42°

**Answer**

**Option A is correct** because one formula for the area of a triangle is $A = \frac{1}{2}ab \sin C$. (Note that $b$ and $a \sin C$ are the lengths of the base and corresponding altitude.) Applying this formula to the given figure yields $A = \frac{1}{2}(8)(15) \sin 42° = 60 \sin 42°$. **Option B is incorrect** because cosine is used instead of sine. **Option C is incorrect** because the $\frac{1}{2}$ in the formula was not used. **Option D is incorrect** because the $\frac{1}{2}$ in the formula was not used and tangent was used instead of sine.

**Competency 010—The teacher understands and solves problems using differential and integral calculus.**

18. **Use the graph below to answer the question that follows.**




The graph of the function $f$ on the interval $0 \le x < d$ is shown above, where $\lim_{x \to d^-} f(x) = +\infty$. For which of the following values of $x$ does $f$ have a removable discontinuity?

A. $a$

B. $b$

C. $c$

D. $d$

**Answer**

**Option B is correct** because at $x = b$ the limit of the function exists but does not equal the value of the function. **Option A is incorrect** because the function is continuous at $x = a$. **Option C is incorrect** because the limit of the function does not exist at $x = c$. **Option D is incorrect** because the function has a vertical asymptote at $x = d$.

# Domain III—Geometry and Measurement

**Competency 011—The teacher understands measurement as a process.**

**Use the information below to answer the question that follows.**

The surface area of a roof is measured in squares of shingles. Each square of shingles covers 100 square feet. However, shingles are sold in bundles and priced per bundle. It takes 3 bundles of shingles to make a square of shingles.

19. A certain roof consists of 2 rectangular sides, each having dimensions 15 feet by 60 feet. Based on the information above, if shingles cost $28.99 per bundle, which of the following represents the total cost of the shingles for the roof?

A. $\frac{(3)(2)(15)(60)(\$28.99)}{100}$

B. $\frac{(2)(15)(60)(\$28.99)(100)}{3}$

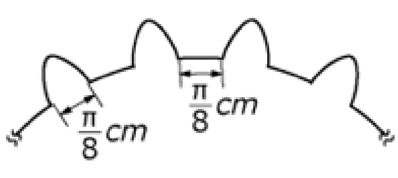
C. $\frac{(2)(15)(60)(\$28.99)}{(3)(100)}$

D. $\frac{(3)(\$28.99)(100)}{(2)(15)(60)}$

**Answer**

**Option A is correct** because the total area of the roof is $(2)(15)(60)\text{ft}^2$. The total cost of the shingles for the roof can be found with the following unit analysis: $(2)(15)(60)\text{ft}^2 \times \frac{1 \text{ square of shingles}}{100\text{ft}^2} \times \frac{3 \text{ bundles}}{1 \text{ square of shingles}} \times \frac{\$28.99}{1 \text{ bundle}}$. So by canceling the units, the total cost is $\frac{(2)(15)(60)(3)(\$28.99)}{100}$, which is equivalent to option A. **Options B, C and D are incorrect** because they are not equivalent to $\frac{(2)(15)(60)(3)(\$28.99)}{100}$.

20. **Use the figure below to answer the question that follows.**




The figure shows a portion of a gear that has cogs evenly spaced around the circumference of a wheel. Each cog is $\frac{\pi}{8}$ centimeters wide, and there is a space of $\frac{\pi}{8}$ centimeters between consecutive cogs. If the diameter of the wheel is 9 centimeters, how many cogs are on the wheel?

A. 12

B. 18

C. 24

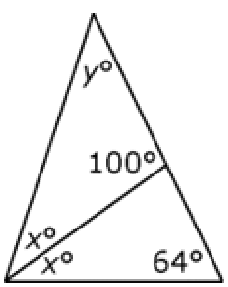
D. 36

**Answer**

**Option D is correct** because the circumference of the wheel is 9π cm and each cog and space requires a total length of $\frac{\pi}{4}$ cm. The number of cogs that will fit around the wheel with spaces in between is $\frac{9\pi}{(\frac{\pi}{4})}$ = 36 cogs. **Option A is incorrect** because 12 cogs and spaces would require a circumference of only 3π cm. **Option B is incorrect** because 18 cogs and spaces would require a circumference of only 4.5π cm. **Option C is incorrect** because 24 cogs and spaces would require a circumference of only 6π cm.

**Competency 012—The teacher understands geometries, in particular Euclidian geometry, as axiomatic systems.**

**Use the figure below to answer the question that follows.**




21. What is the value of *y* in the triangle above?

A. 36

B. 40

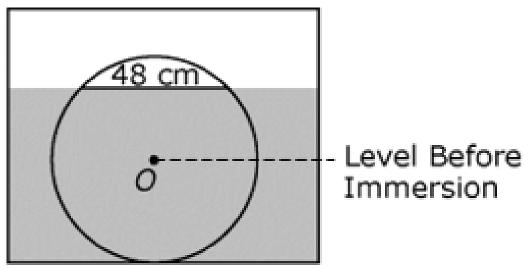
C. 44

D. 48

**Answer**

**Option C is correct.** Based on the lower triangle, $x + 64 = 100$, so $x = 36$. Then, based on the upper triangle, $y + 36 + 100 = 180$, so $y = 44$. **Option A is incorrect** because 36 is the value of *x*, not *y*. **Option B is incorrect** because if $y = 40$, then $x = 40$. **Option D is incorrect** because if $y = 48$, then $x = 32$.

**Use the figure below to answer the question that follows.**




Note: Figure not drawn to scale.

22. A wheel with center *O* and radius 25 cm is immersed in a vat of cleaning solution, as shown in the figure above. The chord of length 48 cm indicates the solution level after the wheel was immersed. The dashed line indicates the solution level before the wheel was immersed. What is the level of the solution in the vat after the wheel has been immersed?

A. 32 cm

B. 33 cm

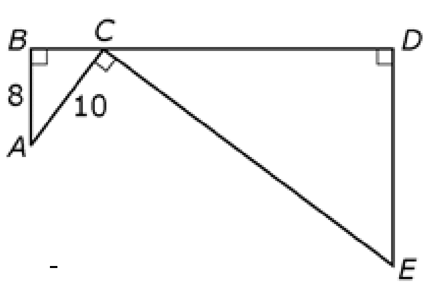
C. 35 cm

D. 37 cm

**Answer**

**Option A is correct.** The level of the solution before immersion is the same as the height of the center of the wheel, which is equal to the radius of the wheel, 25 cm. The height of the solution above the center of the wheel can be found by connecting the center of the wheel to the midpoint and to one endpoint of the chord, forming a right triangle with hypotenuse of length 25 cm and one leg of length 24 cm. The length of the third leg can be found to be 7 cm by the Pythagorean theorem and is equal to the height of the solution above the center of the wheel. So the total height of the water after immersion is 25 + 7 = 32 cm. **Options B, C, and D are incorrect** because the level of the solution after immersion has been shown to be 32 cm.

**Use the figure below to answer the question that follows.**




23. In the figure above, *C* is a point on $\overline{BD}$. Triangles *ABC* and *CDE* are right triangles, and $\overline{AC} \perp \overline{CE}$. If the length of $\overline{BD}$ is 30, what is the length of $\overline{DE}$?

A. 18

B. 20

C. 24

D. 32

**Answer**

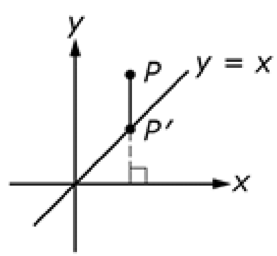
**Option A is correct.** Angle *ACB* must be complementary to both ∠DCE and ∠BAC, so ∠BAC ≅ ∠DCE, and △ABC is similar to △CDE by the AA similarity criterion. Because the triangles are similar, $\frac{AB}{BC} = \frac{CD}{DE}$. By applying the Pythagorean theorem to △ABC, *BC* = 6. Then *CD* = 24, because *BD* = 30 and *BC* = 6. Substituting the known lengths into the proportion yields $\frac{8}{6} = \frac{24}{DE}$, which can be solved to show *DE* = 18. **Option B is incorrect** because doubling the length of segment $\overline{AC}$ does not equal 18. **Option C is incorrect** because 24 is the length of segment $\overline{CD}$, not the length of segment $\overline{DE}$. **Option D is incorrect** because 32 = *AB* + *CD*, which is much greater than the length of segment $\overline{DE}$.

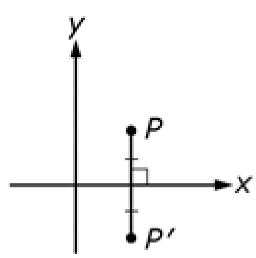
**Competency 014—The teacher understands coordinate, transformational and vector geometry and their connections.**

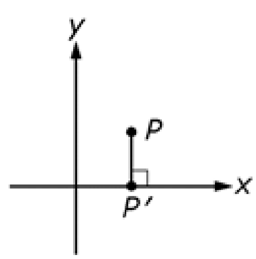
**Use the matrix equation below to answer the question that follows.**

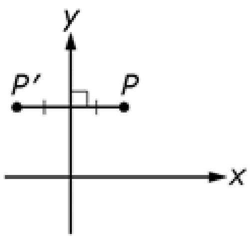
$$\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$$

24. The matrix equation above defines a transformation of the *xy*-plane. Which of the following shows a point *P* and its image *P′* under this transformation?

A. 


B. 


C. 

D. 


**Answer**

**Option A is correct** because multiplying the left side of the given matrix equation gives $\begin{pmatrix} x \\ x \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$. This corresponds to the transformation of a point (*x, y*) to the point (*x, x*), as shown in option A. **Option B is incorrect** because the graph corresponds to the transformation of a point (*x, y*) to the point (*x, –y*). **Option C is incorrect** because the graph corresponds to the transformation of a point (*x, y*) to the point (*x*, 0). **Option D is incorrect** because the graph corresponds to the transformation of a point (*x, y*) to the point (*–x, y*).

## Domain IV—Probability and Statistics

**Competency 015—The teacher understands how to use appropriate graphical and numerical techniques to explore data, characterize patterns and describe departures from patterns.**

**Use the definition below to answer the question that follows.**

For a set of data, a data point is an outlier if it is more than 1.5 times the interquartile range of the data set either above the third quartile or below the first quartile.

The bulb life, in hours, for 27 lightbulbs of the same brand is recorded below.

| | | | | | |
|---|---|---|---|---|---|
| 275 | 400 | 431 | 465 | 480 | 495 |
| 350 | 400 | 436 | 465 | 481 | 595 |
| 360 | 420 | 450 | 470 | 483 | |
| 380 | 425 | 452 | 473 | 490 | |
| 395 | 428 | 460 | 474 | 492 | |

25. Based on the definition above, which of the numbers 275 and 595 is an outlier?

A. Neither 275 nor 595

B. 275 only

C. 595 only

D. Both 275 and 595

**Answer**

**Option B is correct** because the first quartile is 400 and the third quartile is 480, so the interquartile range is 80. By the definition given, any data point that is greater than 1.5(80) = 120 above the third quartile or below the first quartile is considered an outlier. So any data point greater than 600 or less than 280 is an outlier. Thus, 275 is an outlier and 595 is not. **Option A is incorrect** because 275 is an outlier. **Options C and D are incorrect** because 595 is not an outlier.

**Competency 016—The teacher understands concepts and applications of probability.**

26. A computer company employs over 4000 employees, of whom 45% are women. If a focus group of 20 randomly selected employees is to be formed, what is the expected number of men in the focus group?

A. 8

B. 9

C. 11

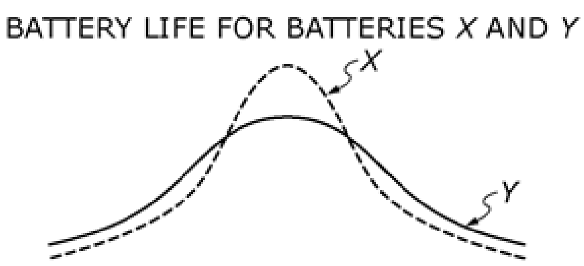
D. 13

**Answer**

**Option C is correct.** If 45% of the employees are women, then 55% of the employees are men. So in a random sample of 20 employees, the expected number of men is 0.55(20) = 11. **Options A, B and D are incorrect** because it has been shown that the expected number of men must be 11.

**Competency 017—The teacher understands the relationships among probability theory, sampling and statistical inference and how statistical inference is used in making and evaluating predictions.**

**Use the graph below to answer the question that follows.**




27. The battery life, in years, for each of two brands of car batteries, *X* and *Y*, is approximately normally distributed, as shown above. Which of the following statements about the mean and standard deviation of battery life for the two distributions is true?

A. The mean battery life for *X* is less than the mean battery life for *Y.*

B. The mean battery life for *X* is greater than the mean battery life for *Y.*

C. The standard deviation of battery life for *X* is less than the standard deviation of battery life for *Y.*

D. The standard deviation of battery life for *X* is greater than the standard deviation of battery life for *Y.*

**Answer**

**Option C is correct** because the curve for battery *X* is steeper and less spread out than the curve for battery *Y*, indicating that the standard deviation for battery *X* is less than that for battery *Y*. **Options A and B are incorrect** because both curves peak at the same value, indicating the same mean. **Option D is incorrect** because the standard deviation for battery *X* is less than that for battery *Y*.

28. To evaluate a new medication that was developed to reduce the occurrence of headaches, a randomized controlled experiment is conducted. One-third of the patients are given the new medication, one-third are given a placebo, and one-third are given nothing. Which of the following is the best example of the placebo effect for this study?

A. People taking the placebo report more headaches than people taking the new medication.

B. People taking the placebo report fewer headaches than people taking the new medication.

C. People taking the placebo report more headaches than people taking nothing.

D. People taking the placebo report fewer headaches than people taking nothing.

**Answer**

**Option D is correct** because the placebo effect refers to a perceived or actual improvement by the group receiving the placebo compared to the group receiving no treatment. **Options A and B are incorrect** because each compares the group receiving the placebo to the group receiving the treatment, not to the group receiving no treatment. **Option C is incorrect** because the placebo effect should show an improvement in the group receiving the placebo.

## Domain V—Mathematical Processes and Perspectives

**Competency 018—The teacher understands mathematical reasoning and problem solving.**

**Use the statement below to answer the question that follows.**

If $x^2$ is even, then $x$ is even.

29. A student is trying to prove that the statement above is true for all integers $x$ by proving its contrapositive. Which of the following procedures should the student follow in order to use this method of proof?

A. Assume that $x^2$ is even, and then deduce that $x$ is even

B. Assume that $x^2$ is not even, and then deduce that $x$ is not even

C. Assume that $x^2$ is even, and then deduce that $x$ is not even

D. Assume that $x$ is not even, and then deduce that $x^2$ is not even

**Answer**

**Option D is correct** because the contrapositive of the given statement is "If $x$ is not even, then $x^2$ is not even." This statement can be proven by assuming that $x$ is not even and deducing that $x^2$ is not even. **Option A is incorrect**

because it describes a method for proving the original statement, but it does not describe the contrapositive. **Option B is incorrect** because it describes a method for proving the inverse of the original statement. **Option C is incorrect** because it does not describe the contrapositive.

**Competency 019—The teacher understands mathematical connections both within and outside of mathematics and how to communicate mathematical ideas and concepts.**

**Use the problem below to answer the question that follows.**

> Working together at their constant rates, hoses *A* and *B* can fill an empty pool in 10 hours. Working alone, it takes hose *B* twice as many hours as hose *A* to fill the pool. How many hours would it take hose *A*, working alone at its constant rate, to fill the pool?

30. In the problem above, if $x$ represents the number of hours it takes hose *A* to fill the pool working alone, which of the following equations correctly models the situation?

A. $\frac{1}{x} + \frac{1}{2x} = \frac{1}{10}$

B. $\frac{1}{x} + \frac{2}{x} = \frac{1}{10}$

C. $\frac{1}{x} + \frac{1}{2x} = 10$

D. $x + 2x = 10$

**Answer**

**Option A is correct** because if hose *A* can fill the empty pool in $x$ hours, then hose *B* can fill the empty pool in $2x$ hours. The fractions of the pool that hoses *A* and *B* can each fill in 1 hour are $\frac{1}{x}$ and $\frac{1}{2x}$ respectively. Working together, it takes the two hoses 10 hours to fill the empty pool, so $\frac{1}{10}$ of the pool can be filled in 1 hour. Thus, $\frac{1}{x} + \frac{1}{2x} = \frac{1}{10}$. **Option B is incorrect** because it takes $2x$ hours for hose *B* to fill the pool, not $\frac{x}{2}$ hours. **Option C is incorrect** because the combined hourly rate equals $\frac{1}{10}$ not 10. **Option D is incorrect** because the total combined time is not equal to the sum of the individual times.

# Domain VI—Mathematical Learning, Instruction and Assessment

**Competency 020—The teacher understands how children learn mathematics and plans, organizes and implements instruction using knowledge of students, subject matter and statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).**

31. Of the following activities involving the quadratic expression $ax^2 + bx + c$, which best exemplifies inquiry-based learning?

A. Students predict how the graph of $y = ax^2 + bx + c$ will be affected by changing the value of $a$, and check their predictions using a graphing calculator.

B. Students solve an equation of the form $ax^2 + bx + c = 0$ by graphing the equation on a graphing calculator.

C. Students derive the quadratic formula by completing the square on the left side of the equation $ax^2 + bx + c = 0$.

D. Students use a function of the form $f(x) = ax^2 + bx + c$ to model a problem involving falling bodies.

**Answer**

**Option A is correct** because inquiry-based learning refers to the practice of allowing students to explore an idea or question on their own. In the described activity, the students use their calculators to explore the effect on the graph of changing the value of *a*. **Options B, C and D are incorrect** because they do not describe an activity in which students explore an idea or question on their own.

**Competency 021—The teacher understands assessment and uses a variety of formal and informal assessment techniques to monitor and guide mathematics instruction and to evaluate student progress.**

32. If a student mistakenly states that $-\frac{1}{2}(-\frac{2}{3}x + \frac{1}{2}) = \frac{1}{3}x + \frac{1}{2}$, it is most likely that the mistake results from a misunderstanding of which of the following?

A. Multiplication of fractions

B. Arithmetic of negative numbers

C. Associative property of multiplication

D. Distributive property of multiplication over addition

**Answer**

**Option D is correct** because the student multiplied only the first term in the parenthesis by $-\frac{1}{2}$, thus making a mistake in the use of the distributive property. **Options A and B are incorrect** because the student multiplied $-\frac{1}{2}$ by $-\frac{2}{3}$ correctly. **Option C is incorrect** because the work does not show an error in the application of the associative property of multiplication.